Lecture 11 Root Locus Design

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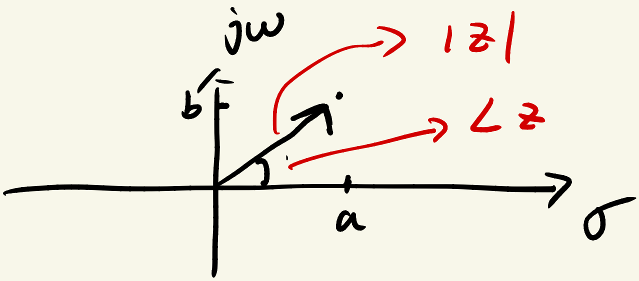
State feedback Design

Computing the Magnitude and Phase of a complex function

A Complex number: $z = a + jb$

Magnitude: $|z| = \sqrt{a^2+b^2}$

Phase: $\angle z = \tan^{-1}\frac{b}{a}$



Product of complex numbers

$$z = z_1 z_2$$

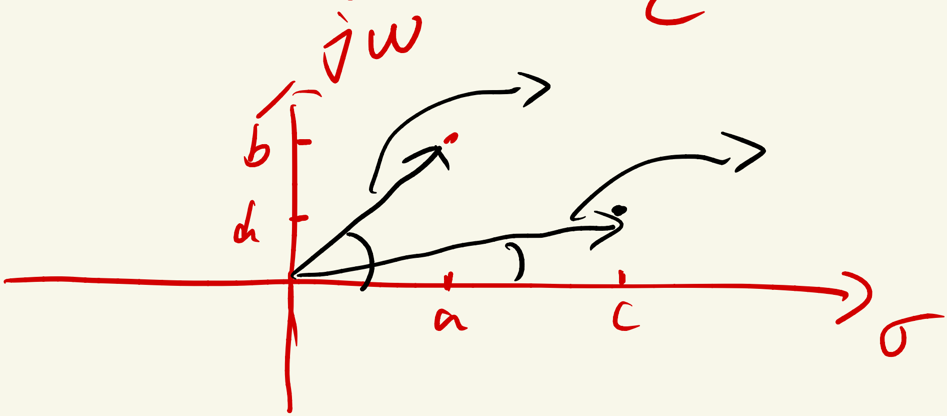
$$z_1 = a + jb \qquad z_2 = c + jd$$

$$|z| = |z_1|\,|z_2|$$

$$= \sqrt{a^2+b^2} \cdot \sqrt{c^2+d^2}$$

$$\angle z = \angle z_1 + \angle z_2$$

$$= \tan^{-1}\frac{b}{a} + \tan^{-1}\frac{d}{c}$$



When dividing complex numbers

$$z = \frac{z_1}{z_2} = \frac{a+jb}{c+jd}$$

$$|z| = \frac{|z_1|}{|z_2|} = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$$

$$\angle z = \angle z_1 - \angle z_2$$

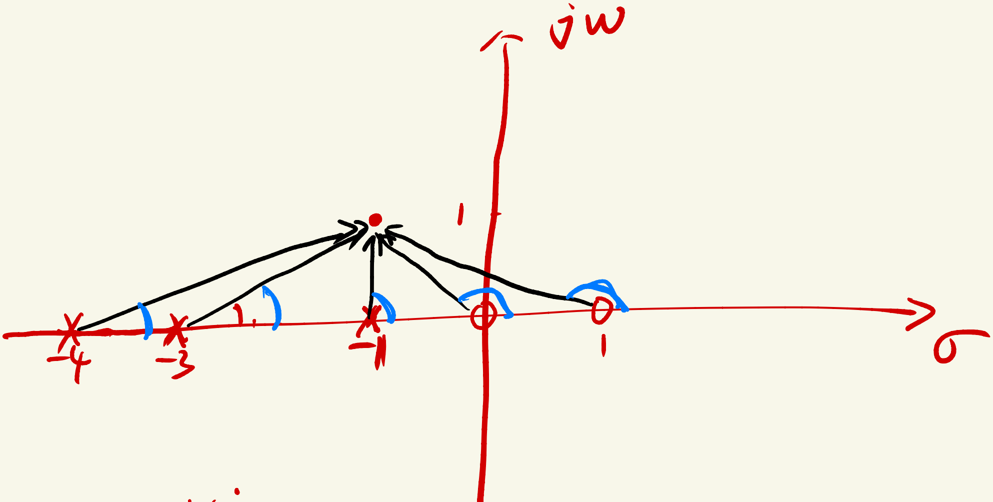
$$= \tan^{-1}\frac{b}{a} - \tan^{-1}\frac{d}{c}$$

Example 1:

Compute the magnitude and phase of

$$G(s) = \frac{s(s-1)}{(s+1)(s+3)(s+4)},$$

at $s = -1+j$



at $s=-1+j$

$$|s+3| = |2+j| = \sqrt{5}$$

$$\angle(s+3) = \angle(2+j) = \tan^{-1}\frac{1}{2} = 26.57^\circ$$

$$G(s)\Big|_{s=-1+j} = G(-1+j) = \frac{(-1+j)(-2+j)}{j\cdot(2+j)\cdot(3+j)}$$

$$|G(s)| = \frac{|(-1+j)|\,|(-2+j)|}{|j|\,|2+j|\,|3+j|}$$

$$= \frac{\sqrt{2}\cdot\sqrt{5}}{1\cdot\sqrt{5}\cdot\sqrt{10}} = \frac{1}{\sqrt{5}}$$

$$\angle G(-1+j) = \angle(-1+j) + \angle(-2+j) - \angle j - \angle(2+j) - \angle(3+j)$$

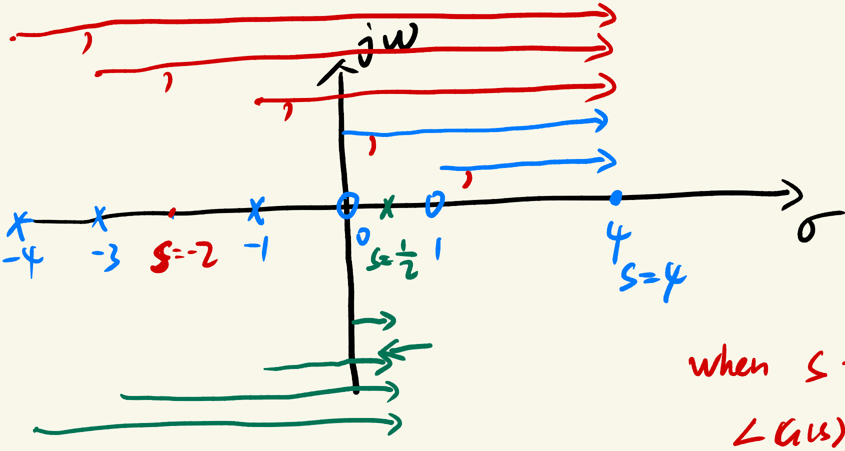
$$= \tan^{-1}(-1) + \tan^{-1}\left(\frac{1}{-2}\right) - \tan^{-1}\frac{1}{0} - \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{3}\right)$$

$$= 153.4^\circ$$

Phase of $G(s)$ on the Real Axis

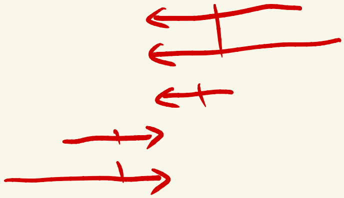
Consider the example

$$G(s) = \frac{s(s-1)}{(s+1)(s+3)(s+4)}$$



when $s=4$

$\angle G(s) = 0$



when $s = \frac{1}{2}$

$\angle G(s) = 180$

$s = -2$

$\angle G(s) = 180$