ELEC 3500
Control Systems
Lecture 5: Block Diagram and Mason's Formula

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A linear time-invariant (LTI) system can be represented in many ways, including:

- differential equation
- state variable form
- transfer function
- impulse response
- block diagram or flow graph

Each description can be converted to the others. We shall see how to represent systems in terms of block diagrams, and how to determine the transfer function of a block diagram system using Mason's Formula.

- **System Interconnections**
  - Series Interconnection
  - Parallel Interconnection
  - Feedback Interconnection
- **Superposition for Linear Systems**
- **MASON’S Formula**

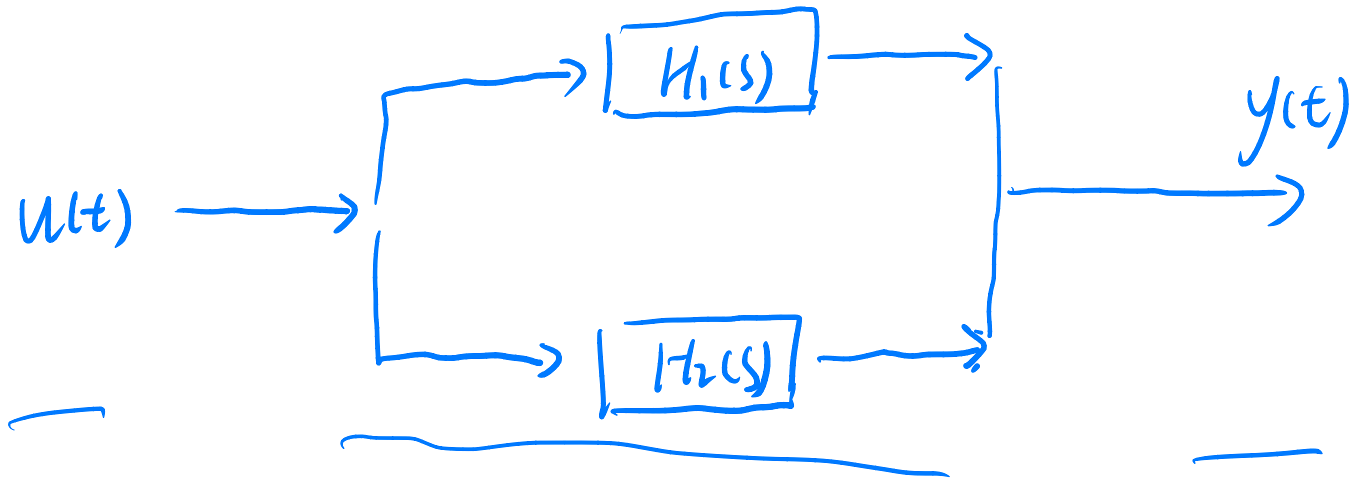
Series Interconnection

$$u(t) \longrightarrow \boxed{H_1(s)} \longrightarrow \boxed{H_2(s)} \longrightarrow y(t)$$

Overall transfer function: $Y(s) = H(s)\,U(s)$

$$H(s) = H_1(s)\,H_2(s)$$

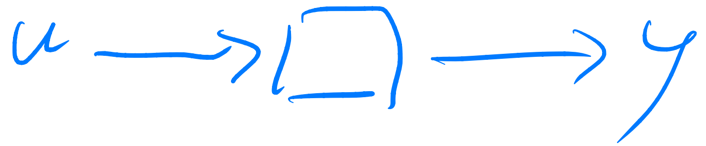
Parallel Interconnection

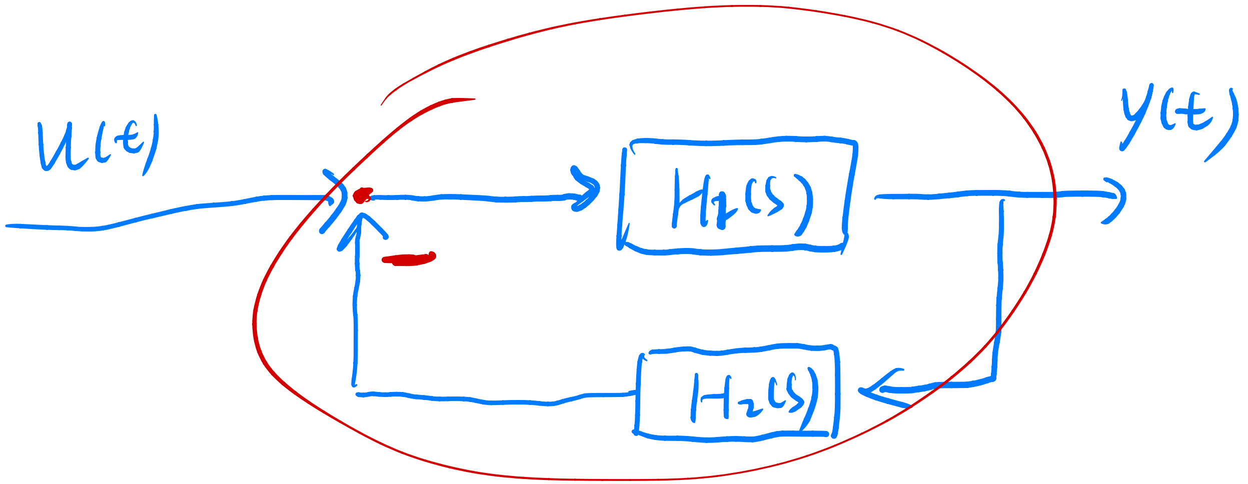


Overall $\quad Y(s) = H(s)\,U(s)$

$$H(s) = H_1(s) + H_2(s)$$

Feedback Interconnection





$$H(s) = \frac{H_2(s)}{1 + H_1(s)\,H_2(s)}$$

Example 1.

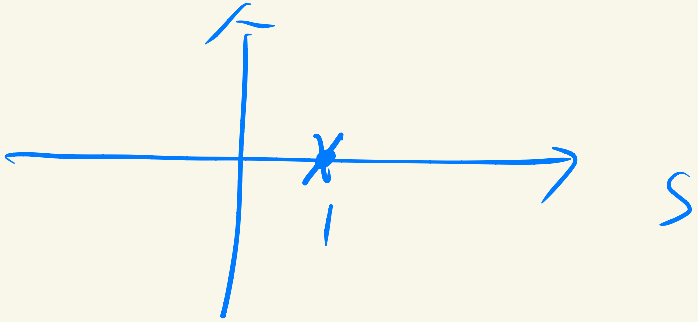
An unstable plant has transfer function

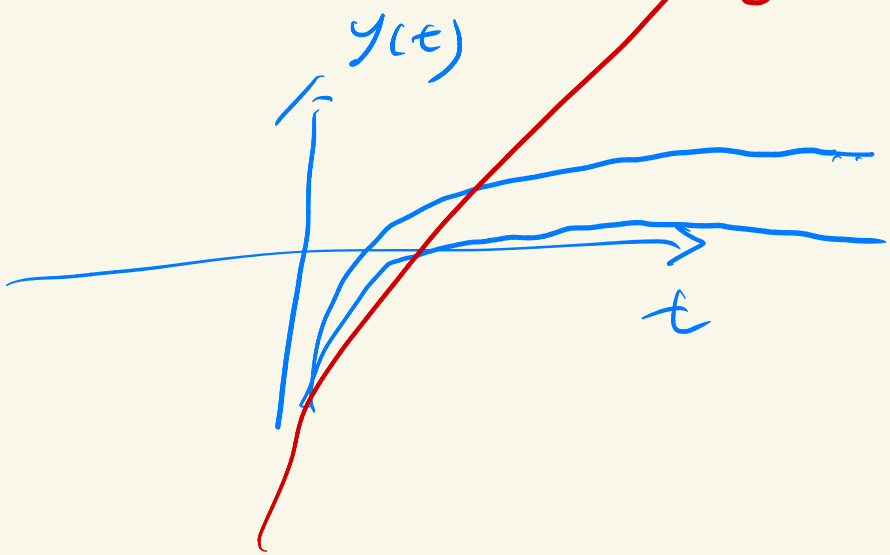
$$u(s) \longrightarrow \boxed{H_1(s)} \longrightarrow y(t)$$

$$H_1(s) = \frac{1}{s-1}$$

pole $s=1 > 0$

not stable

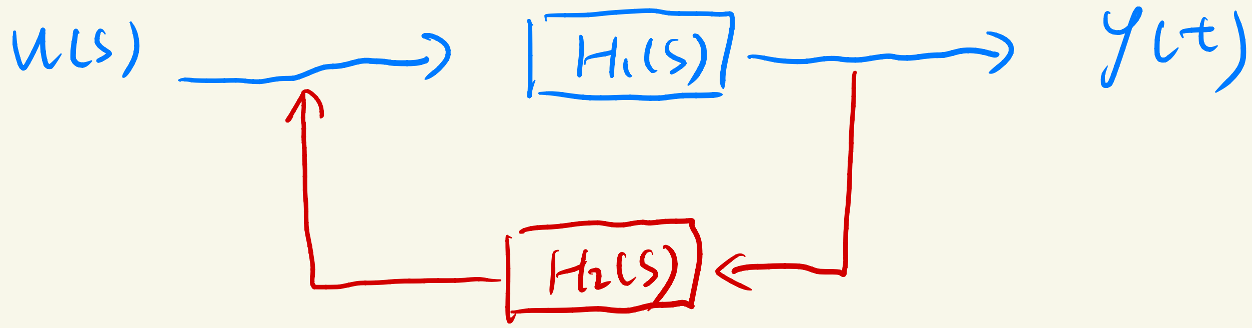




To make $y(t)$ stable,

we use feedback configuration

$$H_2(s) = \frac{10(s+2)}{s+5}$$



$$H(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)} = \frac{\dfrac{1}{s-1}}{1 + \dfrac{1}{s-1} \times \dfrac{10(s+2)}{s+5}}$$

$$= \frac{s+5}{(s-1)(s+5) + 10s + 5}$$

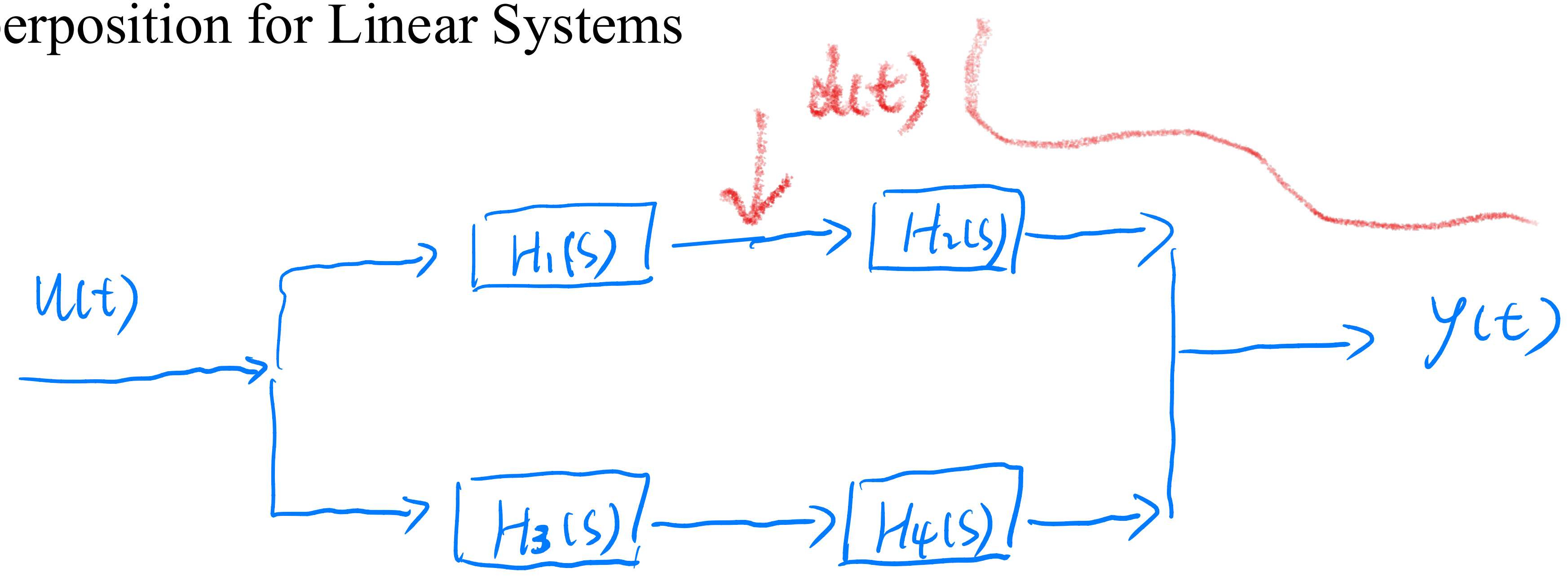
$$= \frac{s+5}{(s+1.7)(s+12.83)}$$

poles

$-1.7, \quad -12.83$

system becomes seable

Superposition for Linear Systems



Overall transfer function

$$H(s) = H_1(s)H_2(s) + H_3(s)H_4(s)$$

Superposition for Linear Systems

From $d(t)$ to $y(t)$

$$Y(s) = H_d(s)\,D(s)$$

$$\downarrow$$

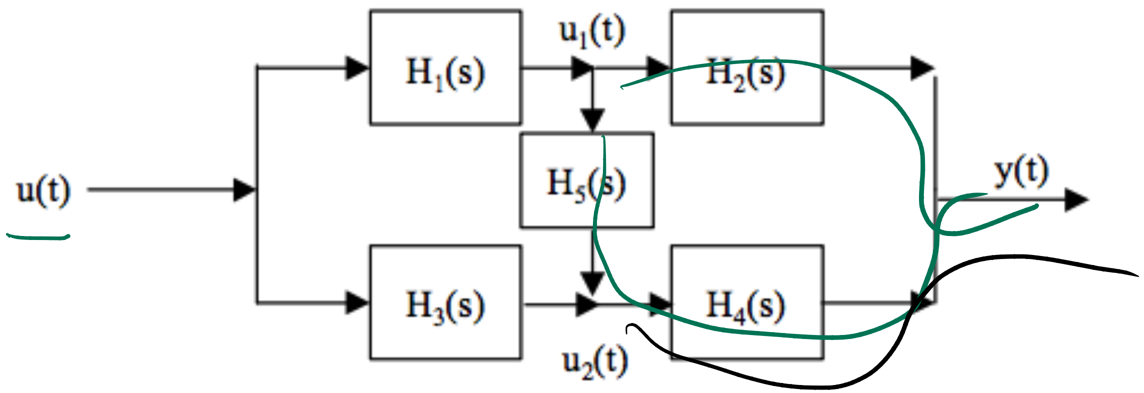
$$H_2(s)$$

From $u(t)$ to $y(t)$: $Y(s) = H_u(s)\,U(s)$

$$H_u(s) = H_1 H_2 + H_3 H_4$$

Overall output : $Y(s) = H_d(s)\,D(s) + H_u(s)\,U(s)$

Example 3- Superposition



From $u_1(t) \to y(t)$

Transfer fun. $H_2(s) + H_4(s) H_5(s)$

From $u_2(t) \to y(t)$

Transfer. fun. $H_4(s)$

From $u(t) \to y(t)$

Three paths

$$H(s) = H_1 H_2 + H_1 H_5 H_4 + H_3 H_4$$

$\downarrow$

Overall transfer function

❑ MASON’S Formula

- Mason's Formula allows one to determine the transfer function of general block diagrams with multiple loops, including feedback loops, and multiple feedforward paths.
- A block diagram consists of *paths* and *loops*. A *loop* is any path where one can go in a circle and return to the beginning point by following arrows in the direction in which they point. Two loops are said to be *disjoint* if they have no elements in common, i.e., if they do not touch.

MASON'S Formula

$$H(s) = \frac{1}{\Delta(s)} \sum_i g_i(s)\Delta_i(s)$$

- g_i represents the transmission along path i.
- $\Delta_i(s)$ is the cofactor of a block diagram with respect to the i-th path and defined as

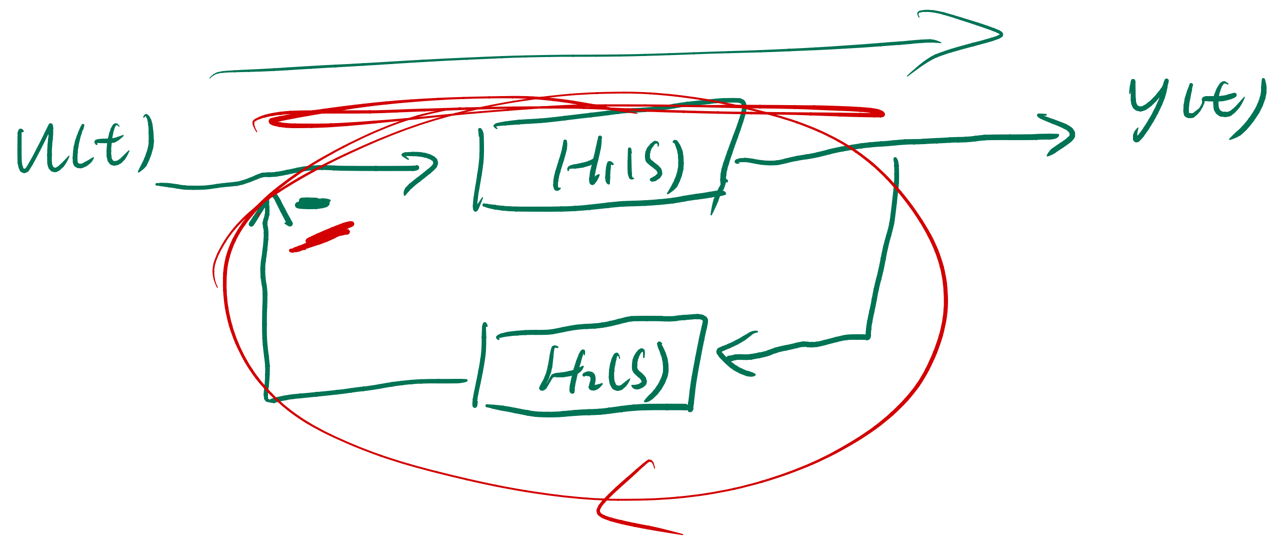
$\Delta_i(s)$=1 - (sum of transmissions of all loops that are disjoint from path i) 0
+ (sum of products of transmissions of all pairs of disjoint loops 0
that are disjoint from path i)
- (sum of products of transmissions of all triples of disjoint loops 0
that are disjoint from path i)
+ ...

- $\Delta(s)$ is the determinant of a block diagram and is defined as

Δ(s)=1 - (sum of transmissions of all loops)
+ (sum of products of transmissions of all pairs of disjoint loops)
- (sum of products of transmissions of all triples of disjoint loops)
- + ...

Note that the i-th cofactor is the same as the determinant, but does not include any loops touching path i.

Example 1- Feedback Connection Using Mason's Formula



Mason's formula $$H(s) = \frac{\sum_i g_i \Delta_i}{\Delta(s)}$$

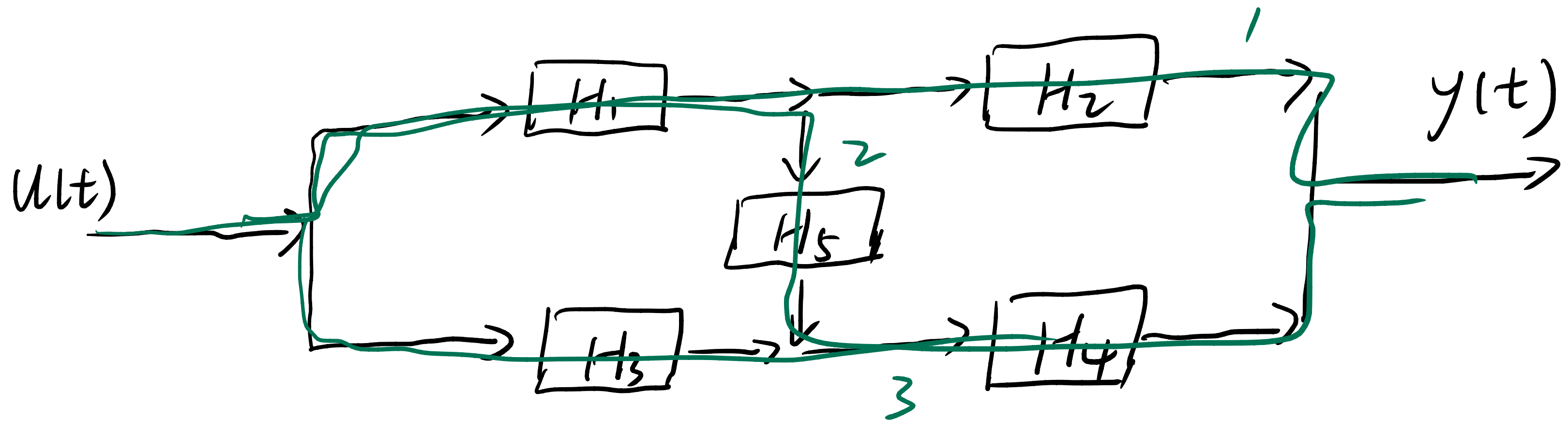
g_i : $H_1(s)$

$\Delta_1 = 1 - 0$

$\Delta = 1 - (-H_1H_2) + 0 - 0$

$$H(s) = \frac{g_1 \Delta_1}{\Delta} = \frac{H_1(s)}{1 + H_1H_2}$$

Example 2- Superposition Using Mason's Formula



Paths : $g_1 = H_1 H_2$

$g_2 = H_1 H_5 H_4$

$g_3 = H_3 H_4$

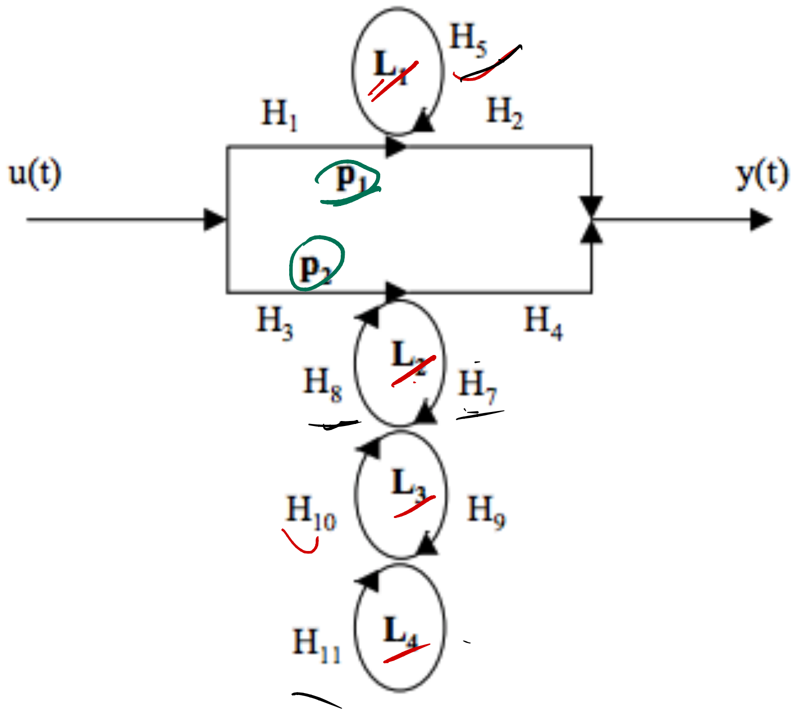
Cofactors : $\Delta_1 = 1$

$\Delta_2 = 1$

$\Delta_3 = 1$

Overall : $H(s) = g_1 + g_2 + g_3$ $\qquad \Delta(s) = 1$

Example 3- Multiloop System



Paths: two paths

$g_1 = H_1 H_2$

$g_2 = H_3 H_4$

Loops: $L_1 \quad L_2 \quad L_3 \quad L_4$

all pairs of disjoint loops:

$(L_1, L_2), (L_1, L_3), (L_1, L_4)$

(L_2, L_4) ✓

triples of disjoint loops

(L_1, L_2, L_4)

$$H(s) = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta(s)}$$

$$g_1 = H_1 H_2$$

$$g_2 = H_3 H_4$$

$$\Delta_1(s) = 1 - (H_7 H_8 + H_9 H_{10} + H_{11}) + (H_7 H_8 H_{11}) - (0)$$

$$\Delta_2(s) = 1 - (H_5 + H_9 H_{10} + H_{11}) + (H_5 H_9 H_{10} + H_5 H_{11}) - (0)$$

$$\Delta(s) = 1 - \left(H_5 + H_7H_8 + H_9H_{10} + H_{11}\right)$$

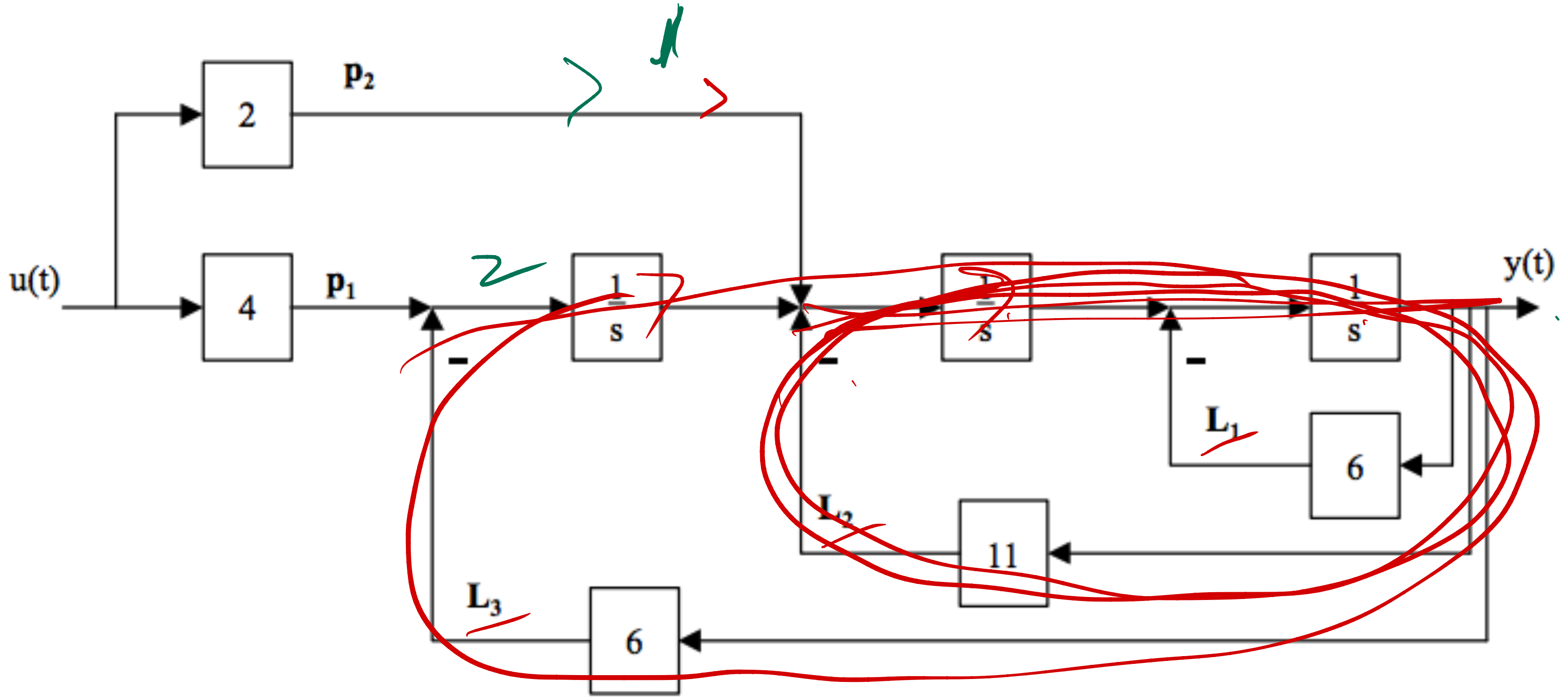
$$+ \left(H_5H_7H_8 + H_5H_9H_{10} + H_5H_{11} + H_7H_8H_{11}\right)$$

$$- \left(H_5H_7H_8H_{11}\right)$$

So,

$$H(s) = \frac{g_1\Delta_1 + g_2\Delta_2}{\Delta(s)}$$

Example 3.1- System in Observable Canonical Form with Single Output



Paths: two paths

$$g_1 = \frac{2}{s^2}$$

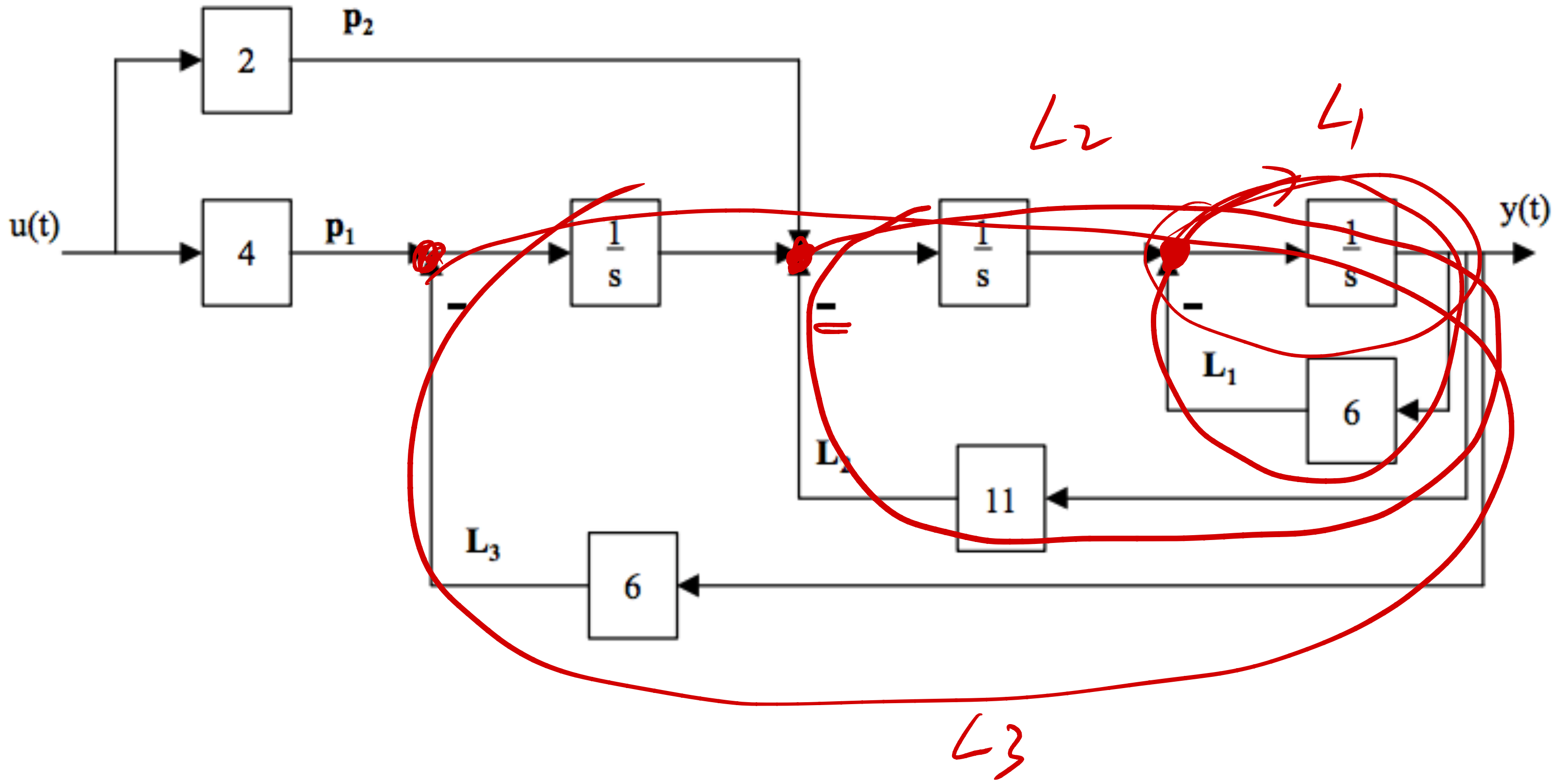
$$g_2 = \frac{4}{s^3}$$

Loops: L_1 L_2 L_3

0 pairs of disjoint loops.

0 triples of disjoint loops.

Example 3.1- System in Observable Canonical Form with Single Output



$$\Delta_1 = 1 - 0 + 0 - 0$$

$$\Delta_2 = 1 - 0 + 0 - 0$$

$$\Delta(s) = 1 - \left(-\frac{6}{s} - \frac{11}{s^2} - \frac{6}{s^3} \right)$$

$$H(s) = \frac{g_1 \Delta_1 + g_2 \Delta_2}{\Delta(s)} = \frac{2s+4}{s^3 + 6s^2 + 11s + 6}$$

Example 3.2- System in Observable Canonical Form with Different Output